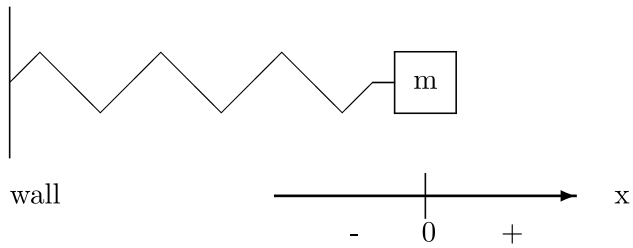
1 The Spring

In the diagram below, a spring is attached to a wall and a mass m. The mass is assumed to glide over a frictionless surface. The spring is ideal in the sense that it is subject to the simple force law given below and has no



mass.

The equilibrium (resting) position of the mass is $x = 0$, and the dispacement x can be either positive or negative. The force exerted by the spring when the mass is moved to position x is

$$\vec{\mathbf{F}} = -kx\hat{x}, \tag{1}$$

where the vector $\hat{x}$ points to the right. Thus, for a positive dispacement the spring exerts a force to the left. For an initial displacement x_o, the total energy of the system consisting of the spring and the mass is entirely potential energy

$$U = \frac{1}{2}kx_o^2. \tag{2}$$

When released, the position of the mass is given by

$$x(t) = x_o \cos\omega t, \tag{3}$$

where $\omega = \sqrt{k/m}$ is the natural frequency. The observed frequency is $\nu = \omega/2\pi$. The total energy remains constant as the sum of the kinetic and potential energies

$$U = \frac{1}{2}mv^2(t) + \frac{1}{2}kx^2(t), \tag{4}$$

where

$$v(t) = \frac{d}{dt}x(t) = -\omega x_o \sin \omega t. \tag{5}$$

Thus, the type of the energy oscillates between all potential and all kinetic

$$U = \frac{1}{2}m\omega^2 x_o^2 \sin^2 \omega t + \frac{1}{2}kx_o^2 \cos^2 \omega t = \frac{1}{2}kx_o^2(\cos^2 \omega t + \sin^2 \omega t). \tag{6}$$

When $x = \pm x_o$ or ωt is a multiple of π then energy is entirely potential, and when $x = 0$ or ωt is an odd multiple of $\pi/2$ the energy is entirely kinetic.